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Superconductor Thermal Stability under the Effect of the Dual-Phase-Lag Conduction Model

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The superconductor thermal stability is investigated under the effect of the dual-phase-lag heat conduction model. Two types of superconductors are considered, Types I and II. It is found that the dual-phase-lag model predicts a wider stable region as compared to the predictions of the parabolic and the hyperbolic heat conduction models. Also, the superconductor thermal stability under the effect of different design, geometrical and operating conditions is studied.

KEY WORDS: dual-phase-lag model; heat conduction; superconductors; thermal stability.

1. INTRODUCTION

Thermal stability is one of the major issues in the design of superconducting devices used in electronic applications and in electric power transmission cables. These devices must be designed in such a way that they are stable against thermal disturbances. Thermal stability denotes a situation where a superconductor can carry the operating current without resistance at all times even if a localized thermal disturbance has been released.

In the literature, numerous researchers [1–11] have investigated the superconductor thermal stability using different superconductor types, geometries, assumptions, applications, operating conditions, and different models. Most previous work has investigated the superconductor stability

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under the effect of the parabolic heat conduction model [1–10], and very few reports have investigated the stability under the effect of the hyperbolic heat conduction model. Based on the authors' knowledge, the superconductor thermal stability under the effect of the dual-phase-lag heat conduction model has not been investigated. This is the objective of the present work. The investigation considers the stability of Type I and II superconductors under different operating, design, and geometrical parameters.

2. ANALYSIS

Consider a thin superconductor of infinite length carrying an electric current as shown schematically in Fig. 1. The superconductor may be of Type I or II. In Type I the superconductor is a non-composite type while in Type II the superconductor consists of a metal matrix and superconducting strands (filaments). The superconductor is cooled using a cooling liquid having convective heat transfer coefficient h. A conductor section of length $2l$ is instantly heated from the heat source up to a temperature T_i, exceeding the critical superconductor temperature T_{c1} at a given current. The temperature field in the normal zone and the quenching process are governed by the energy equation coupled with the dual-phase-lag heat conduction constitutive law. With the assumption of lumped behavior in the radial direction and constant thermal properties, the governing equations are given as

$$C\frac{\partial T}{\partial t}=k\frac{\partial q}{\partial x}-\frac{hP}{A}(T-T_o)+g(t,x) \tag{1}$$

$$q(x,t)+\tau_q\frac{\partial q(x.t)}{\partial t}=-k\frac{\partial T}{\partial x}-k\tau_T\frac{\partial^2 T}{\partial t\,\partial x} \tag{2}$$

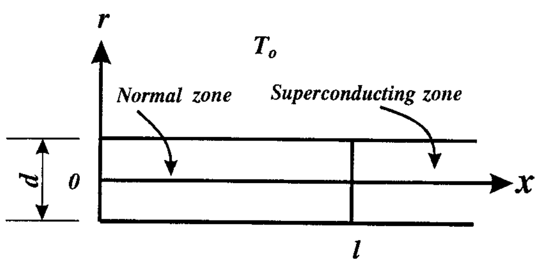

Fig. 1. Schematic diagram for the problem under consideration.

Eliminating q between Eqs. (1) and (2), yields

$$\frac{1}{\alpha}\frac{\partial T}{\partial t}+\frac{\tau_q}{\alpha}\frac{\partial^2 T}{\partial t^2}=\frac{\partial^2 T}{\partial x^2}+\tau_T\frac{\partial^2 T}{\partial x^2}+\tau_T\frac{\partial^3 T}{\partial x^2\,\partial t}-\frac{1}{k}\left[\varphi+\tau_T\frac{\partial\varphi}{\partial t}\right] \tag{3}$$

Where $\varphi=\frac{hP}{A}(T-T_o)$.

In the literature, numerous attention has been paid to the dual-phase-lag heat conduction model as described by Eqs. (1) to (3) [12–16]. The dual-phase-lag describes the temperature response with lagging in the linearized framework while accommodating the first-order effect of τ_q and τ_T. It captures several representative models in heat transfer as special cases. In the absence of the two phase lags, $\tau_q=\tau_T=0$, Eq. (3) reduces to the diffusion equation using Fourier's law. In the absence of phase-lag of the temperature gradient, $\tau_T=0$, Eq. (3) reduces to the wave model. Thus, the two popular models used for describing the macroscopic heat conduction are thus captured in the framework of the dual-phase-lag model under special cases. Also, Eqs. (1) and (3) assume that the system is lumped in the transverse direction of the superconductor. This implies that the temperature variation in the transverse direction of the conductor is insignificant. This condition is valid when the Biot number, defined as hd/k, is much less than 1. This condition is satisfied under most operating conditions, since superconducting materials are of very high thermal conductivity and small radius, while h is of moderate values. The steady capacity of the ohmic heat source is given as for Type I superconductor:

$$\begin{aligned} g(T)&=0 && \text{for}\quad T\leqslant T_{c1}\\ g(T)&=f g_{\max} && \text{for}\quad T\geqslant T_{c1} \end{aligned} \tag{4}$$

and for Type II superconductor:

$$\begin{aligned} g(T)&=0 && \text{for}\quad T\leqslant T_{c1}\\ g(T)&=\frac{T-T_{c1}}{T_c-T_{c1}} && \text{for}\quad T_{c1}<T<T_{c1}\\ g(T)&=g_{\max} && \text{for}\quad T\geqslant T_{c1} \end{aligned} \tag{5}$$

Due to the symmetry of the normal zone, the analysis is limited to the half zone, i.e., to the domain that lies within $x\geqslant 0$. Equation (3) assumes the following initial and boundary conditions

$$
\begin{aligned}
&T(0,x) = T_i \qquad \text{for} \quad 0 < x \leqslant l, \qquad T(0,x) = T_o \qquad \text{for} \quad x > l \\
&\frac{\partial T(0,x)}{\partial t} = 0, \qquad \text{for} \quad 0 \leqslant x \leqslant \infty \\
&\frac{\partial T(0,x)}{\partial t} = 0, \qquad T(t,\infty) = T_o
\end{aligned} \tag{6}
$$

It is more convenient to rewrite Eqs. (3) to (6) using the following dimensionless parameters:

$$
\xi = \frac{x}{2\sqrt{\alpha\tau_q}}, \qquad \beta = \frac{t}{2\tau_q}, \qquad \theta = \frac{T - T_o}{T_c - T_o}
$$

$$
Q = \frac{4\tau_q g(T)}{C(T_c - T_o)}, \qquad H = \frac{4\tau_q hP}{CA}, \qquad R = \frac{\tau_T}{2\tau_q}
$$

As a result, Eq. (3) is reduced to

$$
\frac{\partial^2\theta}{\partial\beta^2} + 2\frac{\partial\theta}{\partial\beta} = \frac{\partial^2\theta}{\partial\xi^2} + R\frac{\partial^3\theta}{\partial\xi^2\,\partial\beta} + \left(Q + \frac{1}{2}\frac{\partial Q}{\partial\beta}\right) - H\left(\theta + \frac{1}{2}\frac{\partial\theta}{\partial\beta}\right) \tag{7}
$$

The heating source in its dimensionless form is rewritten as:

for Type I superconductor:

$$
\begin{aligned}
&Q = 0 \qquad && \text{for} \quad \theta < \theta_{c1} \\
&Q = fG_{\max} \qquad && \text{for} \quad \theta \geqslant \theta_{c1}
\end{aligned} \tag{8}
$$

and for Type II superconductor:

$$
\begin{aligned}
&Q = 0 \qquad && \text{for} \quad \theta \leqslant \theta_{c1} \\
&Q = G_{\max}\frac{\theta - \theta_{c1}}{1 - \theta_{c1}} \qquad && \text{for} \quad \theta_{c1} < \theta < 1 \\
&Q = G_{\max} \qquad && \text{for} \quad \theta \geqslant 1
\end{aligned} \tag{9}
$$

and Eq. (6) is reduced to

$$
\begin{aligned}
&\theta(0,\xi) = B \qquad \text{for} \quad 0 < \xi \leqslant l, \qquad \theta(0,\xi) = 0 \qquad \text{for} \quad \xi > l \\
&\frac{\partial\theta(0,\xi)}{\partial\beta} = 0, \qquad \text{for} \quad 0 \leqslant \xi \leqslant \infty \\
&\frac{\partial\theta(0,\xi)}{\partial\beta} = 0, \qquad \theta(\beta,\infty) = 0
\end{aligned} \tag{10}
$$

2.1. Thermal Stability Criterion

The temperature distribution within the superconductor has the following general form, $\theta = \theta(\beta, \xi, B, R, Q, H, L, f)$. It is obvious that the superconductor maximum temperature occurs at ξ=*0*. If the temperature distribution is stable at this location, then it will be stable elsewhere. This maximum temperature is referred to by θ_1 where $\theta_1 = \theta(\beta, 0, B, R, Q, H, L, f)$. For each combination of B, R, Q, H, L, and f two behaviors can be featured ; (a) θ drops below 1, which means that the normal zone shrinks to zero and the superconductor is stable and (b) θ does not drop below 1, which indicates that the normal zone grows and the superconductor is unstable. We are, in particular, interested in the marginal case when the conditions

$$\theta_1 = 1, \qquad \frac{\partial \theta_1}{\partial \beta} = 0 \tag{11}$$

occur simultaneously. For each combination of B, R, Q, H, L, and f, we may find the critical value of Q, which is referenced to Q_c, where for each $Q \leqslant Q_c$ the two conditions given in Eq. (11) are satisfied. The stability criterion is then: $Q < Q_c$ for collapse (stable) and $Q > Q_c$ for growth (unstable).

2.2. Solution Methodology

The governing equations have been solved numerically by means of the FlexPDE program [17]. FlexPDE is a software tool for the solution of a system of partial differential equations. It offers an integrated solution enviroment, including problem description language, numerical modeling, and graphical output of the solution. FlexPDE uses the powerful finite element method to obtain its numerical solution.

3. RESULTS AND DISCUSSION

Figure 2 shows a comparison between the results of the numerical code used here with that obtained by Bejan and Tien [1] for the superconductor stability using the diffusion heat conduction model. The figure shows the variation of the critical Joule heating source with the disturbance intensity B. It is clear from this figure that the predictions of both models are in good agreement.

Figures 3 and 4 show the transient response of the superconductor maximum temperature θ_1 for different heating sources Q and for Type I

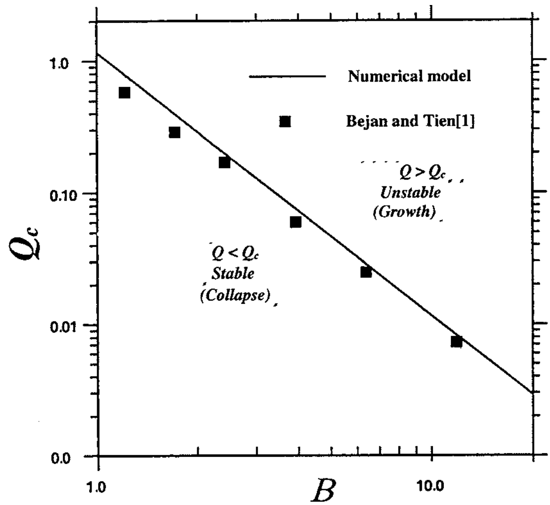

Fig. 2. Comparison between the results obtained in this study and the results reported by Bejan and Tien [1]. ($L = 1.0$, $H = 0.0$, $\xi = 0.0$, $\theta_{c1} = 0.1$, and $f = 0.83$).

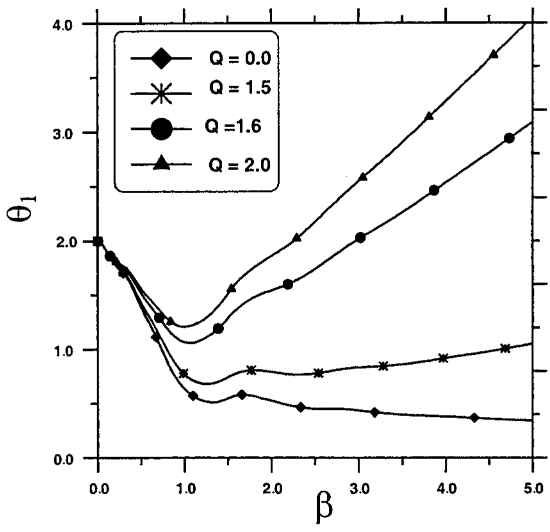

Fig. 3. Effect of dimensionless Joule heating on Type I-superconductor thermal stability based on the dual-phase-lag model. ($R = 0.0$, $L = 1.0$, $\theta_{c1} = 0.1$, $\xi = 0.0$, $B = 2$, $H = 0.0$, and $f = 0.83$).

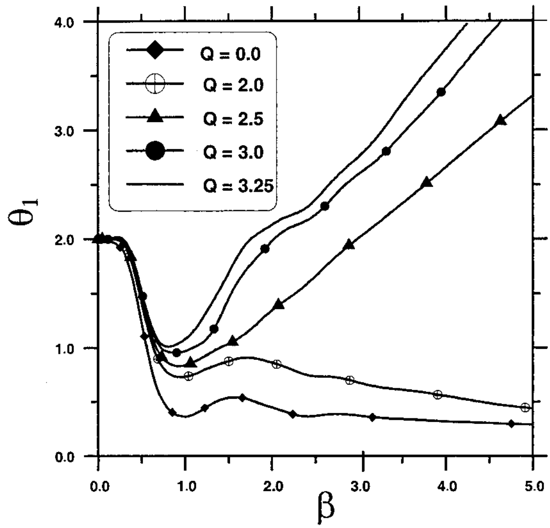

Fig. 4. Effect of dimensionless Joule heating on Type II superconductor based on the dual-phase-lag model. ($L = 1.0$, $R = 5$, $H = 0.0$, $\xi = 0.0$, $B = 2$, and $\theta_{c1} = 0.1$).

and II superconductors. It is clear that the stability collapses as the heating source intensity increases. It is clear that a Type II is more stable than a Type I superconductor. Type II superconductors may sustain heating sources up to $Q = 2$ without destroying the superconductor stability, while Type I is unstable even if the heating source is about 1.5. The reason for this advantage of Type II superconductors is their ability to redistribute the excess current to the critical current from the superconductor into the metal matrix within the normal zone. This, in turn, minimizes the Joule heating effect through the superconductor.

Figure 5 shows the transient response of the superconductor maximum temperature θ_1 at different lateral cooling factors H and for a Type II superconductor. As predicted, it is clear that the stability collapses as the cooling factor decreases.

Figure 6 shows the transient response of the superconductor maximum temperature θ_1 at different initial disturbance lengths L and for Type I superconductors. As predicted, the stability collapses as the disturbance length increases.

Figure 7 shows the transient response of the superconductor maximum temperature θ_1 at a different disturbance duration time τ_i and for Type II

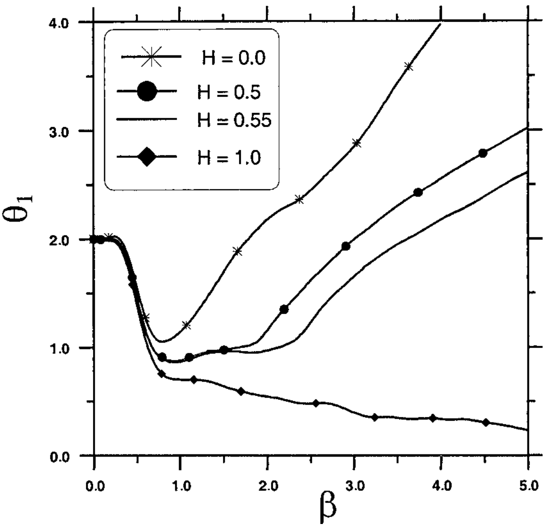

Fig. 5. Effect of dimensionless lateral cooling on Type II-superconductor thermal stability based on the dual-phase-lag model. ($Q = 3.5$, $R = 5.0$, $\xi = 0.0$, $B = 2$, $H = 0.0$, $L = 1.0$, and $\theta_{c1} = 0.1$).

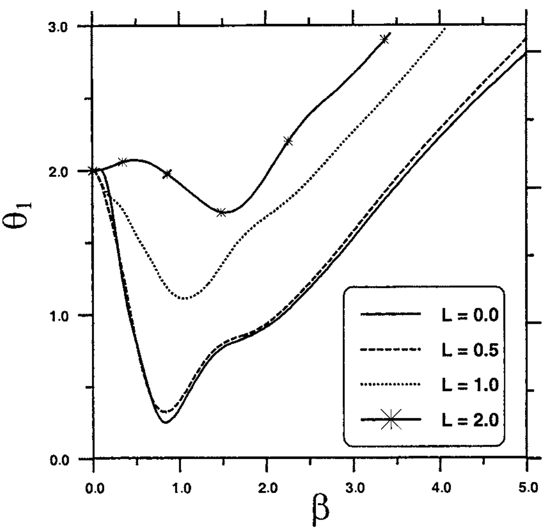

Fig. 6. Effect of dimensionless disturbance length on Type I-superconductor thermal stability based on dual-phase-lag model. ($Q = 2.0$, $R = 5.0$, $\xi = 0.0$, $B = 2$, $f = 0.83$, and $H = 0.0$).

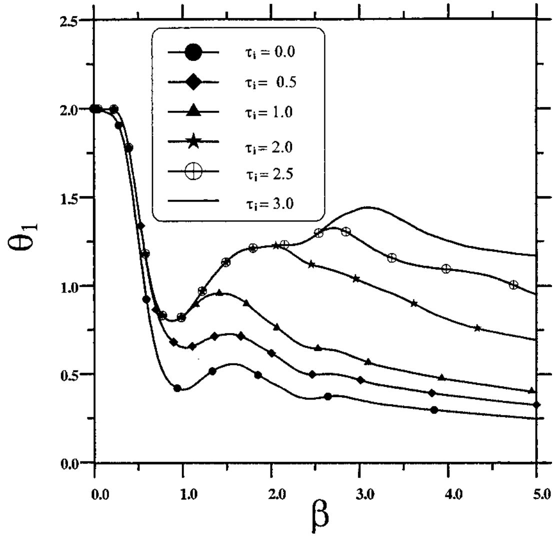

Fig. 7. Effect of the dimensionless disturbance duration time on Type II-superconductor thermal stability based on the dual-phase-lag model. ($L = 1.0$, $Q = 0.5$, $R = 5$, $\xi = 2.0$, $B = 2$, $H = 0.1$, and $\theta_{c1} = 0.1$).

superconductors. The parameter τ_i represents the dimensionless disturbance duration time which is the time within which a fixed imposed initial temperature B is maintained within the normal zone. In other words, τ_i represents the time within which the initial condition in the normal zone remains valid. As predicted, the stability collapses as this time increases.

Figures 8 and 9 show a stability map in terms of the critical Joule heating Q_c and the disturbance intensity B for both superconductor types and using the three heat conduction models. It is clear from this figure that the dual-phase-lag model predicts the widest stable region as compared to the predictions of the other two heat conduction models. However, there is no clear trend for the deviations between the predictions of the wave and diffusion models. Also, the three models predict a linear relation between Q_c and B. The deviations among the predictions of the three models vanish as the lateral cooling factor H increases. It is obvious that as B increases, the conductor ability to sustain Q_c while remaining stable, decreases. Also, it is clear from these two figures that the stability region for Type II superconductors is wider than that of Type 1. The deviations among the three models vanish as B increases. The effect of the lateral cooling factor H is insignificant at small values of B.

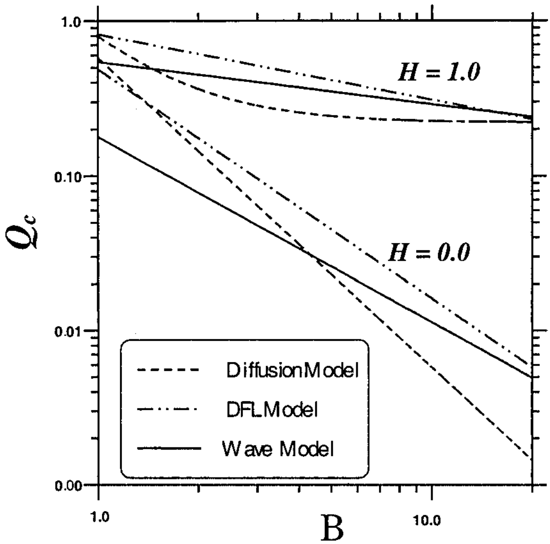

Fig. 8. Stability criterion for Type I superconductor subjected to stepwise type disturbance, based on the three different heat conduction models. ($L = 1.0$, $R = 5$, $\xi = 0.0$, $\theta_{c1} = 0.1$, and $f = 0.83$).

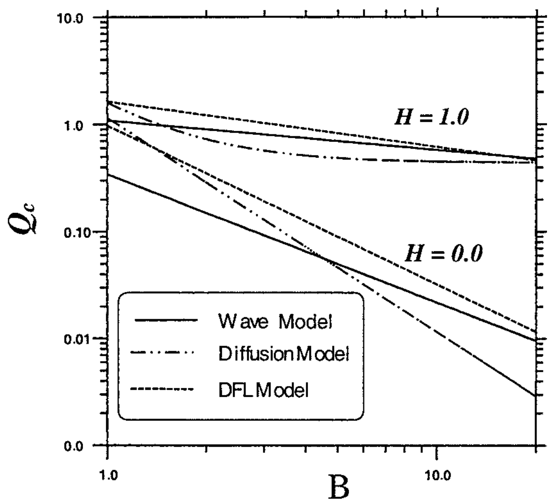

Fig. 9. Stability criterion for Type II superconductor subjected to stepwise type disturbance, based on the three different heat conduction models. ($L = 1.0$, $R = 5$, $\xi = 0.0$, and $\theta_{c1} = 0.1$).

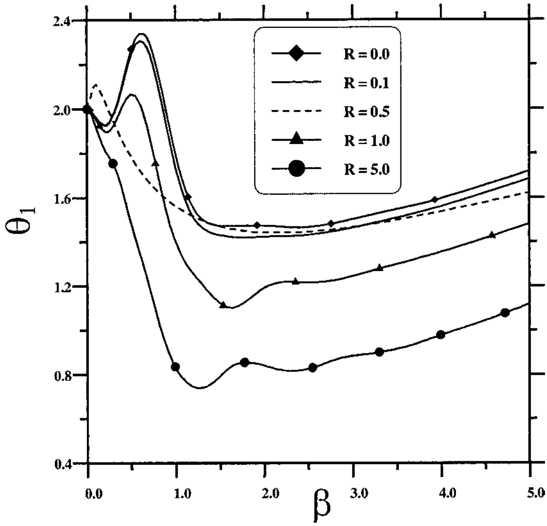

Fig. 10. Comparison between dimensionless maximum temperature-time history of Type I superconductor based on the three macroscopic heat conduction models. ($Q = 1$, $L = 1.0$, $\xi = 1.0$, $B = 2$, $H = 0.10$, and $f = 0.83$).

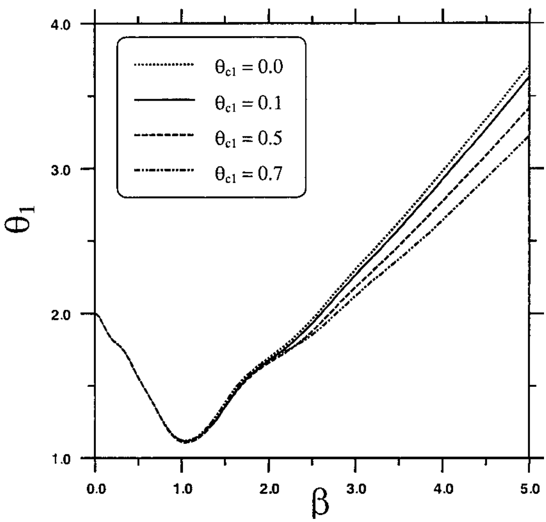

Fig. 11. Effect of dimensionless current sharing temperature on Type II-superconductor thermal stability based on the dual-phase-lag model. ($L = 1.0$, $Q = 1$, $R = 5.0$, $\xi = 0.0$, $B = 2$, and $H = 0.0$).

Figure 10 shows the transient response of the superconductor maximum temperature θ_1 at different relaxation time ratios R and for Type I superconductors. It is clear that the stability collapses as R decreases. As R decreases, the predictions of the dual-phase-lag model approach that of the diffusion model. As mentioned previously, the dual-phase-lag model predicts a wider stability region than the diffusion model.

Figure 11 shows the transient response of the superconductor maximum temperature θ_1 at different current sharing temperatures θ_{c1} and for Type II superconductors. It is clear that as the current sharing temperature decreases, the stability collapses. However, this effect is insignificant. This is obvious since as θ_{c1} increases, the superconductor can sustain a localized thermal disturbance having a higher temperature before entering the normal zone.

4. CONCLUDING REMARKS

The superconductor thermal stability is investigated under the effect of the dual-phase-lag heat conduction model. The parameters which are found to affect the superconductor thermal stability are the disturbance intensity B, volume fraction of the stabilizer f, initial disturbance length L, lateral cooling factor H, Joule heating source Q, relaxation time ratio R, current sharing temperature θ_{c1} and the disturbance duration time τ_i. It is found that the dual-phase-lag model predicts a wider stable region as compared to the predictions of the parabolic and hyperbolic heat conduction models. As predicted, the study shows that a Type II superconductor is more stable than for Type I. The superconductor stability improves as L, B, τ_i, and Q decrease and as H, R, and θ_{c1} increase. However, the effect of the current sharing on the superconductor stability is insignificant.

NOMENCLATURE

A	Conductor cross sectional area, m^2
A_m	Matrix cross sectional area, m^2
B	Dimensionless disturbance intensity, $\dfrac{T_i - T_o}{T_c - T_o}$
C	Heat capacity, $J \cdot m^{-3} \cdot K^{-1}$
d	Conductor diameter, m
f	Volume fraction of the stabilizer in conductor
g	Joule heating, $W \cdot m^{-3}$

$g_{\max}$	maximum Joule heating with the whole current in the stabilizer, $\dfrac{\rho J^2}{p}$, $\mathrm{W\cdot m^{-3}}$
$G_{\max}$	Dimensionless maximum Joule heating, $\dfrac{4\tau_q g_{\max}}{C(T_c - T_o)}$
h	Convective heat transfer coefficient, $\mathrm{W\cdot m^{-2}\cdot K^{-1}}$
H	Lateral cooling factor, $\dfrac{4\tau_q hP}{CA}$
J	Current density, $\mathrm{A\cdot m^{-2}}$
k	Thermal conductivity of conductor, $\mathrm{W\cdot m^{-1}\cdot K^{-1}}$
$2l$	Length of conductor subjected to heat disturbances, m
P	Conductor perimeter, m
q	Conduction heat flux, $\mathrm{W\cdot m^{-2}}$
Q	Dimensionless Joule heating source, $\dfrac{4\tau_q g(T)}{C(T_c - T_o)}$
R	Relaxation time ratio $\tau_T/(2\tau_q)$
t	Time, s
T	Temperature, K
T_c	Critical temperature, K
T_{c1}	Current sharing temperature, K
T_i	Initial temperature, K
T_o	Ambient temperature, K
x	Spatial coordinate, m

Greek Symbols

α	Thermal diffusivity,
β	Dimensionless time, $\dfrac{t}{2\tau_q}$
θ	Dimensionless temperature, $\dfrac{T - T_o}{T_c - T_o}$
θ_{c1}	Dimensionless current sharing temperature, $\dfrac{T - T_o}{T_c - T_o}$
θ_1	Dimensionless maximum temperature, $\dfrac{T(t,0) - T_o}{T_c - T_o}$
ξ	Dimensionless axial location, $\dfrac{x}{2\sqrt{\alpha\tau_q}}$
ρ_o	Stabilizer electrical resistivity, Ω

τ_i Dimensionless duration time , $\dfrac{t_i}{2\tau_q}$

τ_q Relaxation time of heat flux, s

τ_T relaxation time of temperature gradient, s

Subscripts

i Initial

sc Sharing current

max Maximum

o Ambient

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